

The validity of the Cattell-Horn-Carroll model on the intraindividual approach

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ABSTRACT

The Cattell-Horn-Carroll (CHC) model is considered the state-of-the-art of the psychometric tradition about intelligence. However, researchers of the dynamic systems field argue that the interindividual variation applied by psychometrics on intelligence field can produce inferences about the population but not about an individual. The present study investigated the validity of the CHC model at the level of the individual through the intraindividual approach. A dynamic factor analysis was employed in order to identify the factor structure of one individual scores on nine tests of the Higher-Order Cognitive Factor Battery, throughout 90 measurement occasions. Those tests measure, in the population level, three second order abilities and the general factor of the CHC model. Only the general intelligence factor was identified. Ultimately, the CHC model did not present validity to the assessed person. Implications for intelligence theories and measurement are discussed.

KEYWORDS: intelligence; CHC model, intraindividual approach, ergodic theorems

THERE IS A CONFLICT between two ways to study the human being and they are explained by the nomothetic and idiographic approaches. Whereas the idiographic approach tries to study singularity and particularities, the nomothetic approach studies groups and uses statistics and quantitative analysis to make generalizations. It can be observed that in the “fight” between these two approaches, the nomothetic approach is winning and has been the mainstream in psychology. One of the reasons for this is because the idiographic approach is many times seen as exclusively a clinical area, distant from the rigor and precision required by the science. However, there are studies, such as the one from Robinson (2011), that deny this vision and show that the idiographic versus nomothetic debate was misinterpreted. As pointed:

“Idiographic knowledge aims at describing and explaining particular phenomena Nomothetic knowledge, on the other hand, has the aim of finding generalities that are common to a class of particulars and deriving theories or laws to account for these generalities” (Robinson, 2011, p. 1).

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As stated by Robinson, the idiographic approach comes from the Wundtian idea that science needs to develop methodologies that adequately understand individual cases, or particular phenomena. So, in this sense it drives theorizations.

On the other side, it also serves as a way to test the theories, following the refutation processes that were formally proposed by Popper (1972). As Robinson's study points, Allport made a mistake when he equated the idiographic approach with a specific research method, i.e. the study of individuals, and the nomothetic approach with another specific research method: the study of groups and populations. In this way, the idiographic approach could be viewed as an enemy of science because it would be concerned only about singularity and particularities. However, this is not true, and Allport's dichotomization was a misconception of Windelband's concepts (Robinson, 2011). If the nomothetic approach aims for generality, this is only possible through the idiographic approach. The latter provides what Popper (1972) called “negative singular cases” providing to the former the refutability that are essential in science. It is only from the idiographic approach that we can see the “black swan” and refute the theory that was developed considering the other approach. Therefore, these two approaches should be seen as complementary and not opposite.

Many of the traditionally used methodologies in psychology, such as exploratory and confirmatory factor analysis, come from the general linear model that usually estimates groups and not indi-

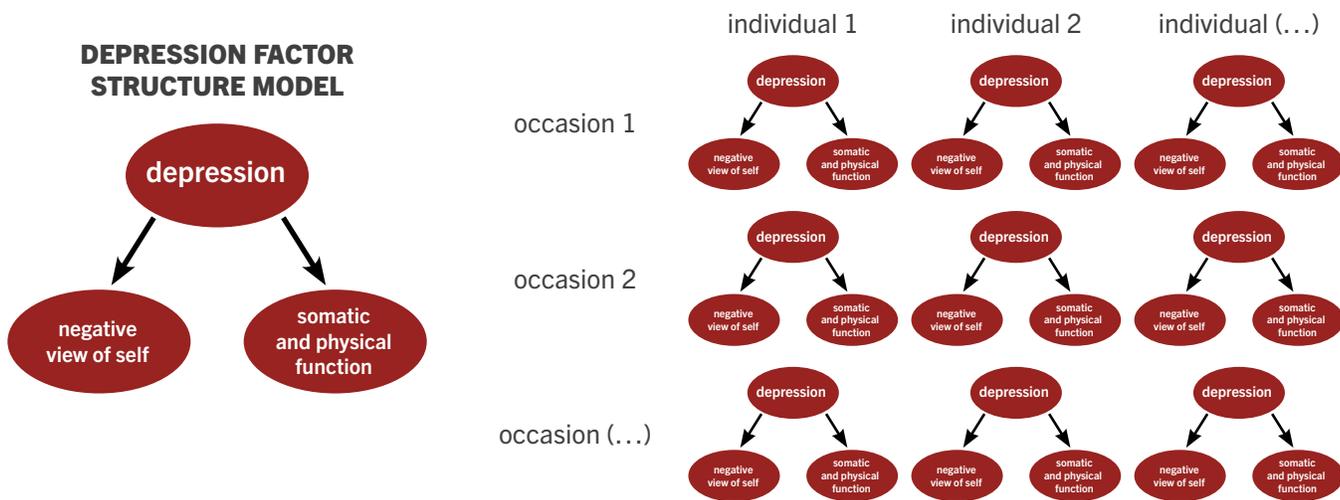


Figure 1. example of the depression factor model and the representation of the homogeneity and ergodicity conditions.

viduals. These methodologies are used to investigate the structure, organization and distribution of psychological constructs in the population level. They are tools from the nomothetic approach, called interindividual approach because their statistics are based in the differences between individuals (Molenaar, 2007A). Although providing important information for the understanding of psychological functioning in the population, the interindividual approach normally cannot provide information about how individuals function (Molenaar, Sinclair, Rovine, Ram & Corneal, 2009). The inappropriateness to directly transpose population information to individual was mathematically proved through the ergodic theorems.

The ergodic theorems, which come from the mathematical theory of ergodicity, are mathematical-statistical models that were first developed in the 1930's to study dynamic systems, due mainly to the work of Henri Poincaré, George Birkhoff and John Von Neumann (Ugalde, 2007). The ergodic theorems define two necessary and sufficient conditions that allow the generalization of knowledge from the interindividual structure observed in the population to the individuals: homogeneity and stationarity. The homogeneity criterion requires that all the individuals from the population have the same statistical structure and parameters (Molenaar, 2007). According to the stationarity criterion, this statistical structure cannot vary throughout the time. For example, consider a specific construct, such as depression. Suppose that it is composed by one higher-order factor (depression) and two specific factors: negative view of the self, and somatic and physical function (Figure 1). In order to follow the homogeneity criterion, all the individuals from the population should have the same factor structure and should follow the stationarity criteria. This factor structure could not vary across multiple measurement occasions. Most of the psychological processes violate both conditions, and therefore are considered non-ergodic processes. For this reason, it would not be possible to transpose the population data directly to the individual. Taking all this into consideration, Molenaar proposes an intraindividual approach.

The behavior and psychological characteristics of a person are usually non-ergodic (Nesselroade & Molenaar, 2010). Despite the "bad news", both intraindividual and interindividual variation can be analyzed at the same time. Raymond Cattell (1952), in the early 50's, had already proposed a model to study the psychological variables that involved the analysis of the differences between individuals, called *R*-technique (interindividual), as well as the analysis of a single subject throughout the time, called *P*-technique (intraindividual) as can be seen in Figure 2. He explains that all the experimental designs in psychology have three components—individuals, time and variables, and depending on how they are combined, a different technique is used. The *R*-technique measures one or more variables, in several individuals, during a single occasion or a few occasions and allows identifying common factors in the population. The *P*-technique, by the other side, measures one or more variables in a single subject, during several occasions. However, the technique that

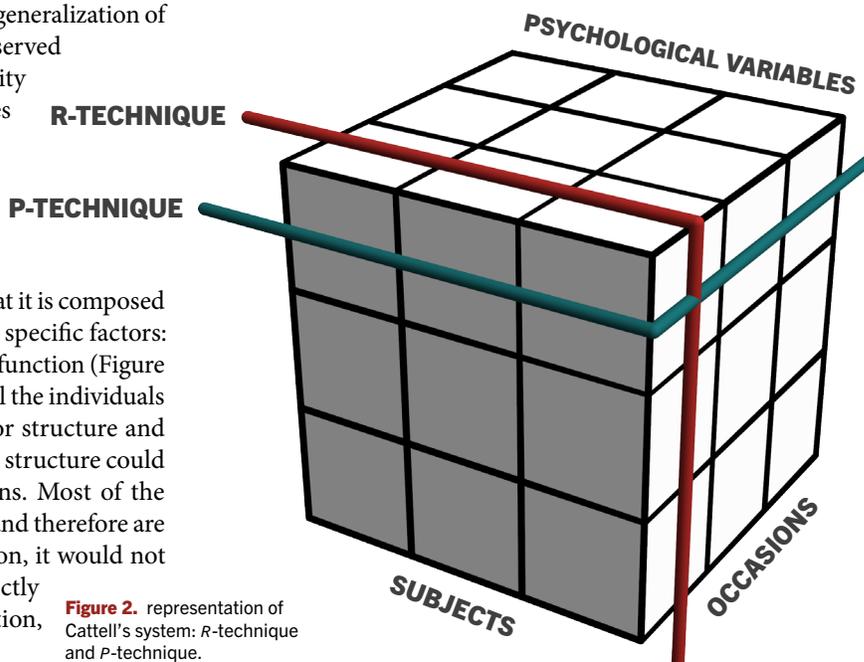


Figure 2. representation of Cattell's system: *R*-technique and *P*-technique.

Cattell proposed for an intraindividual analysis was the same used for the interindividual analysis: the *P*-factor analysis and the traditional factor analysis do not consider the score dependency across the time.

The *P*-technique was criticized because it is very similar to the traditional factor analysis and does not consider the score dependency across the time. If a person responds to an instrument 100 times, a correlation matrix will be produced from the 100 raw scores of the person data. That correlation matrix is similar to the traditional matrix used on factor analysis. It considers that the answers are independent and that the previous behaviors do not influence the next, which is rarely true (Ram, Brose, & Molenaar, 2013). Theoretically, previous answers predict or are related to the next and the *P*-technique does not incorporate this prerogative. An approach that aggregates the conditionality between the achievements is the dynamic factor analysis or the time series factor analysis. These involve a correlation matrix plus a time series analysis. Different strategies of data arrange and estimation are capable to adequately account for the relationships between responses over time. One of these is the Toeplitz matrix, for example.

Ram, Brose and Molenaar (2013) synthesize the main differences between the Cattell's *P*-technique factor model, time series factor model and the dynamic factor model in a very comprehensive way. The authors state that the objective of the *P*-technique is to describe relations among multiple responses of *P*-data, i.e. data collected in multiple occasions in one or more variables, in order to discover the structure underlying the responses or to test hypothesis regarding the day-to-day variation observed. However, as pointed before, since repeated measurements obtained from the same person are generally related, a key assumption required by factor analysis will probably be violated: the independency of the observations (Ram, Brose, & Molenaar, 2013). Ram, Brose and Molenaar (2013) point that in the years following the development of the *P*-technique factor model, a number of alternatives emerged to account for the relationship between the variables, for example the autoregression and moving average time series' models. In 1985 Peter Molenaar introduced the dynamic factor analysis as an alternative to *P*-technique factor model and to the time series models, since it enables two things: "deal with the independence violations and provide a framework for modeling the dynamic nature of ongoing processes" (Ram, Brose, & Molenaar, 2013, p.3). In the dynamic factor model, the multivariate state of an individual at any time is given by concurrent influences and past states (Ram, Brose, & Molenaar, 2013).

The psychometric approach to intelligence

In the early twentieth century, Spearman (1904) introduced a new perspective in psychology. He developed a key instrument to analyze data in psychology, called factor analysis, enabling the empirical research of psychological constructs through analysis of a correlation matrix on cognitive tests. Spearman (1904) identified one factor that explained the common variance among all IQ tests. Such factor was called *g* or general intelligence factor (Spearman, 1904). The specific variance not explained by *g* received the name of specific factor or *s*.

A body of studies was conducted throughout the years and divergent evidences were found. (Spearrit, 1996). In order to solve the problem of divergent evidences, John Carroll (1993) published in the early 1990's a meta-analysis study that included the main researches about intelligence conducted in the last 80 years. Carroll's findings suggested that intelligence has three levels or strata. The higher stratum, level 3, is composed by the general intelligence factor. The intermediate stratum, level 2, consists of eight broad cognitive abilities, while the basic stratum, level 1, has more than 50 specialized abilities.

In the late 1990's, McGrew and Flanagan (1998) proposed the integration of the Cattell-Horn-Carroll models, creating the CHC (Cattell-Horn-Carroll) model. This model consists of a multidimensional view of the intelligence, with three cognitive levels: the general factor (3rd level), 10 broad cognitive abilities (2nd level) and more than 70 specialized abilities (1st level). The CHC model has been validated in several papers, all around the world. In the Brazilian literature, for example, Gomes and Borges (2007), Gomes (2010) and Wechsler and Schelini (2006), found evidence supporting the CHC model. However, all of the validity studies of the CHC use the interindividual approach. Taking the ergodic theorems as reference, it is not possible to state that the three levels of the cognitive architecture found in the population are also present at the individual level without validation at the individual level.

The present study aims to analyze the validity of the CHC model in one individual, using the intraindividual approach. In order to do that, nine intelligence tests from the Higher-Order Cognitive Factor Battery (BAFACALO—Gomes, 2010) were administered to a single subject, on 90 different occasions. Using the time series and single case study design, we seek to verify if the CHC model is valid to explain the intraindividual variation of the scores from one individual over time. We expect to find, through the use of dynamic factor analysis, three latent variables (fluid intelligence—*Gf*, processing speed—*Gs* and crystalized intelligence—*Gc*) and at least a moderate correlation among them, indicating the presence of the general intelligence factor (*g*). In relation to the data of this study, the CHC model predicts that a general intelligence factor must be encountered. This general factor must explain three lower-order factors (fluid intelligence—*Gf*, processing speed—*Gs*, and crystalized intelligence—*Gc*). These three specific factors should explain, respectively, its specific marker tests. As pointed, the CHC model defines a hierarchical relationship between the cognitive abilities and it is expected that the commented hierarchy will be present. On the contrary, the CHC model could be refuted.

» METHOD

Participant

The subject of this study was a 23 year old student, who graduated in biological sciences at the *Pontifícia Universidade Católica do Paraná, Brazil*. At the time he participated in the study, he was studying ergodic theorems, but he did not have any knowledge about the psychometric models of intelligence or any previous contact with the tests used in this study. The participant is one of the co-authors of this study, but great care was taken to prevent his previous knowledge from affecting his performance in the

tests, creating bias in the results. The participant had not had any contact with the tests, before they were administered to him during the trial. Furthermore, at the moment that the participant responded the tests, he did not know anything about the psychometric intelligence models and the question of this study. He did all the tests in a blind way.

Instruments

Nine tests of the Higher-Order Cognitive Factor Battery were used. The Battery of Higher-Order Cognitive Factors or the BAFACALO's project, was developed by Gomes (2005, 2010), and was based on the Educational Testing Service's Kit of Factor-Referenced Cognitive Tests (1976). In the theoretical domain, the BAFACALO battery was developed to assess the general intelligence factor (g), plus six broad abilities presented in Carroll's three stratum theory (Carroll, 1993) and the CHC model (McGrew, Keith, Flanagan, & Vanderwood, 1997) in high school students. These broad abilities are: fluid intelligence (Gf) (Gomes & Borges, 2009A), crystalized intelligence (Gc) (Gomes, 2012), short-term memory (Gsm) (Gomes, 2011), broad visual perception (Gv) (2009B), fluency (Gr) and broad cognitive speediness (Gs).

Evidences show that the Higher-Order Cognitive Factor Battery is able to measure g and six broad cognitive abilities of the second level from the CHC model (McGrew et al., 1997). The structure of the battery was investigated by Gomes (2010) using Exploratory Factor Analysis (EFA) and Structural Equation Modeling (SEM). Each factor retention criterion used resulted in a different factor structure. The parallel analysis identified three factors, while the Kaiser criterion suggested a four factor structure. Both, the scree plot and the maximum likelihood approach, suggested a six factor structure. The second-order general factor was identified in every solution by a second-order EFA. Each structure pointed by the EFA result was tested via SEM. The three broad factors' model with a second order general factor presented the worst fit ($\chi^2/df = 3.02$, CFI = .87, RMSEA = .08). The four broad factors' model with a second order general factor presented a χ^2/df of 2.45, a CFI of .91 and a RMSEA of .07, while the six broad factors' model with a second order g presented the best fit ($\chi^2/df = 1.39$, CFI = .98, RMSEA = .04).

Despite the sample of the original study presenting a broad variety of socio economic status (SES) levels, which is present in the Brazilian population of high school students, the sample was not intended to be representative of the Brazilian high school students. On the contrary, the sample is a convenient sample composed by 292 Brazilian high-school students from one public school (53.40% girls, 46.60% boys) of Belo Horizonte, Minas Gerais, Brazil. The majority of the sample was composed by girls, reflecting the demographic characteristic of the Brazilian population. Their age ranged from 14 to 20 years old (Mean $M = 15.71$, Standard Deviation $SD = 1.15$). Most of the participants had a monthly household income varying from R\$1,751 to R\$3,500 Reais. In order to recruit participants for this study, the school principal sent an invitation letter to all the students of the school, with the research purpose, the name and contact of the research team, as well as the dates of data collection. The chief of the school's Psychology Department visited every class reinforcing the Principal's invitation to participate in the study, and answered every question raised by the students.

Those interested in being part of the study were contacted by the researchers and signed a consent form, and confirmed to be at the school in the scheduled testing days. From the 320 students enrolled in the school, 91.25% accepted being part of the study, and answered the tests. It was not possible to know about sampling bias from the 8.75% of students that did not accept to participate of the study. The school only disposed data to the researchers about the students that accepted to participate of the research and signed the consent form. It is worth mentioning that Ethics Committees in Brazil does not allow incentives in researches involving the human being, so no incentive was given to the students.

All tests have Cronbach's alpha above .70, and also present structural, divergent, convergent, predictive and incremental validity (Gomes & Borges, 2009A, 2009B; Gomes, 2010; Gomes, 2011; Gomes, 2012). The first three tests listed measure the fluid intelligence (Gf), the three following tests measure the crystalized intelligence (Gc) and the last three tests measure the processing speed (Gs).

Inductive test (I). It consists of 12 items with an execution time limit of 14 minutes. Each item is composed by five groups of 4 letters. Among the 5 groups there are four groups that represent the same pattern, i.e. the letters are organized according to the same rule. The respondent has to identify the group of letters that does not follow the pattern and mark it with an X.

Logical reasoning test (LR): This test consists of 30 items and the time limit for its execution is 24 minutes. Each item consists of a conclusion based on two abstract logical premises, with no relationship to the real world. The respondent has to indicate if the logical conclusion is appropriate or inappropriate.

General reasoning test (GR): It consists of 15 items and the time limit for its execution is 18 minutes. Each item is composed by a mathematical-logical problem, with a statement and a space to solve it. The respondent has to interpret the statement, solve the problem and choose one of the five multiple-choice answers.

Verbal comprehension test 1 (V_1): It consists of 24 items and the time limit for its execution is 6 minutes. Each item is composed of one reference word and five multiple-choice words. The respondent must identify the word which best approximates, in terms of meaning, to the reference word and mark it with an X.

Verbal comprehension test 2 (V_2): It consists of 18 items and the time limit for its execution is five minutes. Each item is composed of one reference word and five multiple-choice words. The respondent must identify the word which best approximates, in terms of meaning, to the reference word and mark it with an X.

Verbal comprehension test 3 (V_3): It consists of 18 items and the time limit for its execution is five minutes. Each item is composed of one word of reference and four multiple-choice words. The respondent must identify the word which best approximates, in terms of meaning, to the reference word and mark it with an X.

Perceptual speed test (P_1): The test consists of 410 words, whose 50 words begin with the letter "a" and the time limit is two minutes. The task is to mark all the words with the letter "a" in the given time Perceptual Speed Test 2 (P_2). The test has 48 items and its time limit is one and a half minute. Each item has the following task: to compare pairs composed by several digits and to identify if they are equal or different.

Perceptual speed test 2 (P_2): The test contains 48 items and its time limit is one minute and a half. Each item has a template figure and five options; only one option is identical to the template. The respondent must identify the figure that corresponds to the template.

Procedures

During approximately three months, the same nine intelligence tests from the Higher-Order Cognitive Factor Battery (BAFACALO) were administered to the same subject in 90 different occasions. The participant had contact with the tests only at the moment of the administration. After the last administration moment, the tests were scored by raw score sum. The score in each test corresponded to the number of correct answers of the participant. The scores were registered in Excel spreadsheet and each line corresponded to the score of the participant in a specific moment.

Data analysis

In temporal series, the performance is influenced by the previous performance. For that reason, the traditional exploratory factor analysis methods are not suitable to analyze this kind of data, as it assumes score errors' independence. The software DYFA2.03 from Browne and Zhang (2005) was used to analyze the scores produced during the 90 moments of the test administration. This software performs the dynamic factor analysis, what is appropriate to investigate data from temporal series. The dynamic factor analysis used the following basic equations (Ram, Brose, & Molenaar, 2013):

$$y_t = \Lambda\eta(t) + \varepsilon(t) \tag{1}$$

$$\eta(t) = B_1\eta(t - 1) + B_2\eta(t - 2) + \dots + B_s\eta(t - s) + \zeta(t), \tag{2}$$

where y_t is a vector of the observable variables indexed by time ($t = 1, 2, \dots, T$), Λ is the $p \times q$ factor loading matrix, $\eta(t)$ is a q -variate time series of latent factor scores and $\varepsilon(t)$ is the specific error plus measurement error time series. In equation 2, The $\eta(t)$ is modeled as a function of prior weighted (B_1 to B_s) latent states from $\eta(t - 1)$ to $\eta(t - s)$. As pointed by Ram, Brose and Molenaar (2013), "present time "disturbances" are then introduced as a q -variate set of latent "innovations," $\zeta(t)$, and residual (measurement + specific) errors, $\varepsilon(t)$, the latter of which may be correlated across occasions." The common factors follow the VARMA process (p, q), generating a manifest-stationary temporal series. According to Browne and Zhang (2005), there is no likelihood function available for lagged covariance matrix, therefore the goodness of fit indexes are not applicable. The estimation method used is a simple discrepancy function of the minimum ordinary squares.

DYFA provides a series of rotation procedures of the factor solutions. The rotation used in this study was the Crawford-quartimax Ferguson, equivalent to quartimin, a solution that enables the identified oblique factors to correlate. The dynamic exploratory factor analysis used in this study was VARMA (1, 1) with a three

maximum lag, the number of occasions was 90, the number of individuals and common factors was one. The estimation approach used was the two stages method and the observable variables were the tests $P_1, P_1, P_2, P_3, GR, LR, I, V_1, V_2, V_3$, described previously.

» RESULTS

The participant's performance showed variance in all the tests, which is an essential condition for the intraindividual variation analysis. Table 1 shows the means, standard-deviations and the maximum performance in the nine tests employed in this study. The P_3 test was the easiest test, because the average of correct answers was 99.5%. The GR test was the second easiest. The most difficult test was P_2 , since the average achievement was 53.58%. The test with the smallest performance variance was P_3 , with a standard deviation of 2.06%. On the other hand, the P_1 test was the one that showed the highest performance variance, with a standard-deviation of 16.72%. There was a ceiling effect, where all items were passed by the participant, on the P_1, P_3 , and GR tests.

The temporal series of the participant's performance during the 90 measuring occasions are shown on Figure 3. The P_3 test showed a very high growth rate between the first and second occasion. After the second administration moment the performance remained relatively constant, reaching an asymptote around the twentieth measuring occasion. The test I showed a similar pattern.

The V_1 and V_2 tests showed a high initial growth rate with some small decreases during that time. After that, a plateau-type performance is observed, followed by a sudden increase in the performance and a new plateau. The V_3 test showed a similar pattern to the V_1 and V_2 tests. However, the decreases present in the initial growth are more substantial. There is also a stronger change in the performance after the first plateau. The GR test shows a pattern relatively similar to the one found in V_2 . However, GR does not present a plateau type pattern followed by a sudden change. In GR it is possible to observe an asymptote around the twentieth occasion. The performance in P_2 test was variable during all the time, with improvements and decreases over the whole period. Overall, there was a gradual increase in performance. The performance in P_1 showed an increase more constant than in the test P_2 , appearing to be more gradual and characteristic of a logistic growth curve. The performance in LR showed a relatively erratic pattern, similar to the test P_2 , at the beginning of the time series. However, the performance growth became more stable and gradual from the twenty-fifth occasion of measurement, and in the fiftieth occasion, it reached the asymptote.

Table 2 shows the correlations between the observable variables during lag zero. As can be seen, in general, the correlations are moderate to high suggesting the presence of a general factor. A superficial examination did not seem to indicate that the specific groupings P_1, P_2 and P_3 (group of tests to measure G_s), LR, I and GR (group of tests to measure G_f) and V_1, V_2 and V_3 (group of tests to measure G_c) have intragroup correlations higher than the

Table 1. means, standard-deviations and maximum of the observable variables

tests	M (%)	SD (%)	max (%)
P_1	87.26%	16.72%	100.00%
P_2	53.58%	6.94%	68.75%
P_3	99.51%	2.06%	100.00%
V_1	81.38%	6.83%	91.67%
V_2	73.33%	5.56%	83.33%
V_3	85.00%	5.39%	94.44%
GR	95.78%	11.65%	100.00%
LR	80.93%	9.81%	90.00%
I	89.91%	5.34%	91.67%

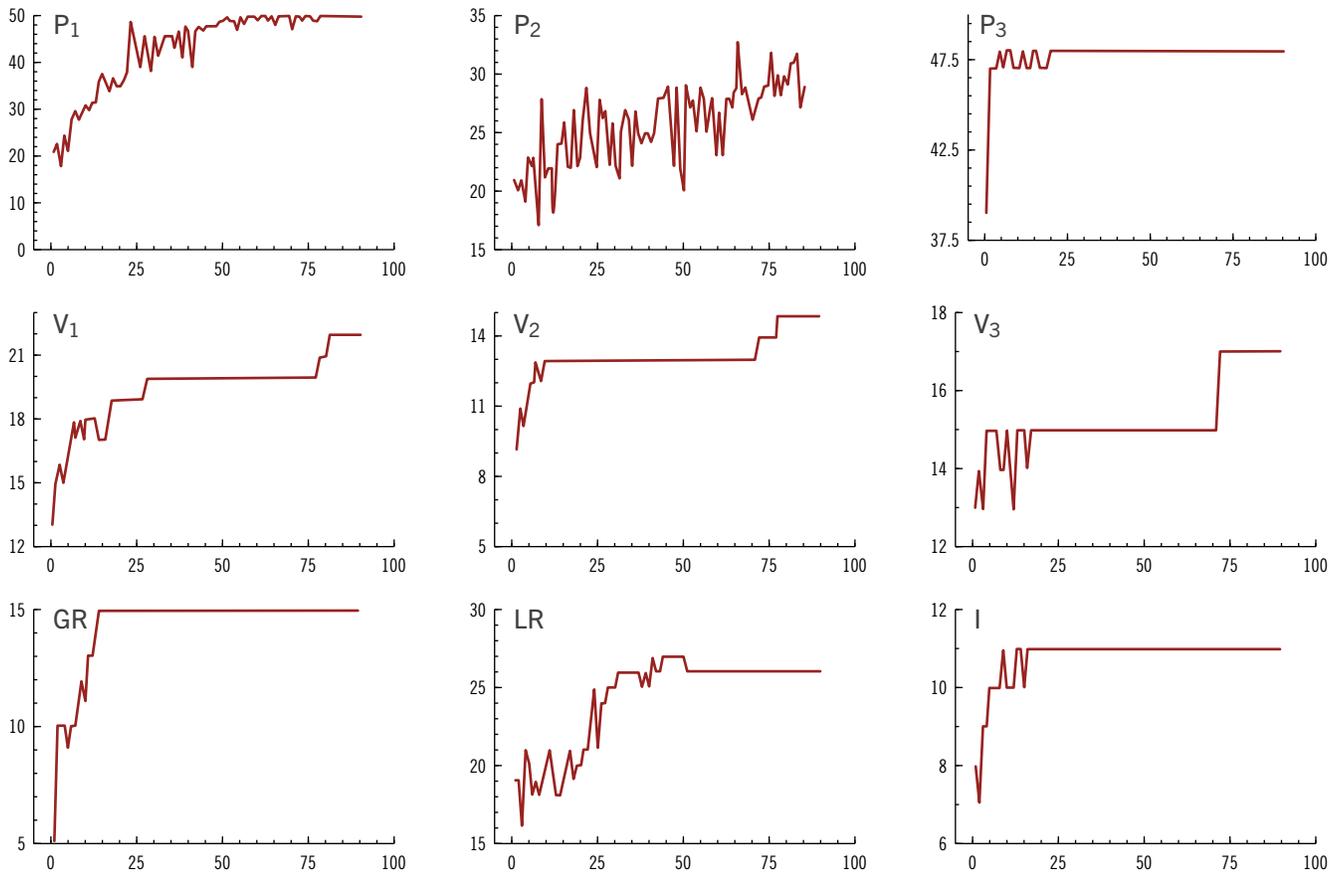


Figure 3. temporal series of the participant's performance in the 90 measuring occasions

extra-group correlations, possibly indicating that the three broad skills, Gf, Gs and Gc from the CHC model, are not identified in the individual examined.

The Cattell's scree test showed that a factor solution with a single factor was adequate. Figure 4 shows the eigenvalues in decreasing order and a line indicating which eigenvalues can be considered noise and should not be retained. This line crosses the second eigenvalue up to the sixth eigenvalue and it shows that only the first eigenvalue should be retained.

An exploratory dynamic factor analysis, with VARMA (1, 1) process, was performed using three lags to estimate the solution.

Table 2. correlation matrix of the observable variables during the lag zero

	P ₁	P ₂	P ₃	V ₁	V ₂	V ₃	GR	LR	I
P ₁	1.00	0.64	0.50	0.87	0.66	0.59	0.80	0.89	0.73
P ₂	0.64	1.00	0.29	0.65	0.62	0.57	0.48	0.57	0.48
P ₃	0.50	0.29	1.00	0.59	0.54	0.34	0.69	0.42	0.59
V ₁	0.87	0.65	0.59	1.00	0.84	0.69	0.78	0.80	0.76
V ₂	0.66	0.62	0.54	0.84	1.00	0.86	0.64	0.51	0.64
V ₃	0.59	0.57	0.34	0.69	0.86	1.00	0.43	0.49	0.42
GR	0.80	0.48	0.69	0.78	0.64	0.43	1.00	0.65	0.85
LR	0.89	0.57	0.42	0.80	0.51	0.49	0.65	1.00	0.58
I	0.73	0.48	0.59	0.76	0.64	0.42	0.85	0.58	1.00

In order to have a satisfactory exploratory dynamic factor solution, two conditions should be met. The first one is the identification of the VARMA model and the second one is the stationarity condition. The software DYFA 2.03 has two indicators, one for each condition. The solution of one general factor met both: stationarity of 0.91, smaller than 1 (necessary limit) and the average VARMA moving weight of 0.09, smaller than 1 (necessary limit).

Figure 5 shows how much the participant's performance in each test is correlated to his previous performance in the same test. This indicator is given by the ACF (auto-correlation factor) and it can be observed that, in the majority of the tests, the auto-correlation decrease as the intervals (lags) between one measurement and another measurement progress. This fact indicates that the previous performance is affected by the immediately previous performance and that the effect of this influence decreases along the occasions. Figure 5 also allows us to conclude that the choice for three lags was sufficient to identify the correlation matrix. In the axes of each test in Figure 5, there is a pair of horizontal lines below and above the x-axis, indicating how many lags are required for the correlation matrix to be captured by the factor solution. The PACF (partial auto-correlation factor) allows such identification, as it indicates if the auto-correlations residue is statistically significant according to the lags chosen. None of the partial auto-correlations is statistically significant after the third lag.

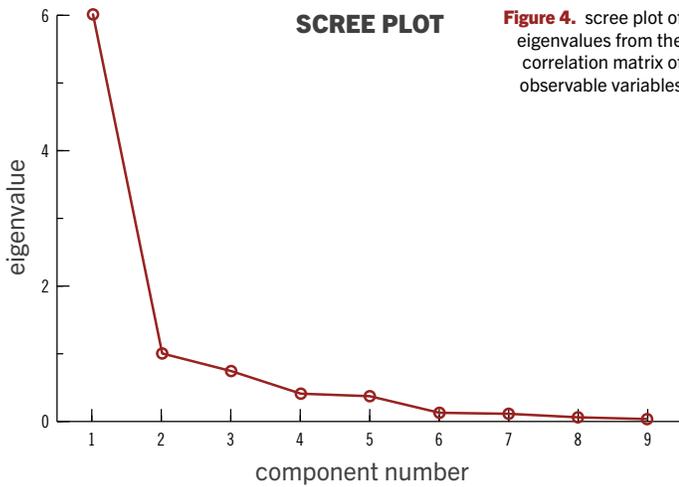


Figure 4. scree plot of eigenvalues from the correlation matrix of observable variables

(lag 1) of the general factor are highly correlated (.89). In turn, two measurement occasions of the general factor with another measurement time point between them (lag 2) also show a high correlation (.81).

» **DISCUSSION**

The aim of this study was to investigate the validity of the CHC model to identify the cognitive architecture of a specific individual, given the limitations of using population data to make inferences about individuals (Molenaar, 2007A, 2007B). So, the intraindividual approach analyzed the variance in the cognitive performance of one subject over 90 measurement occasions. The results are contrary to the presence of the expected three broad abilities (Gf, Gs and Gc). A single latent variable was identified indicating that the Spearman’s model (1904) is the model which best explains the participant’s performance, by the general factor (g) and the specific factor (s) of each test administered. The factor loadings found in this study varied from moderate-high to high, indicating that the general factor is of great importance to explain the variance of all tests administered. Another considerable part is explained by a specific factor in each test (Spearman’s s factor), which is, at the same time, mixed with a portion of the measurement error.

The factor loadings of the tests in relation to the general factor were from moderate-high to high ($V_1 = .97$; $P_1 = .92$, $GR = .85$; $V_2 = .84$; $I = .80$; $LR = .78$; $V_3 = .68$; $P_2 = .67$; $P_3 = .61$, in descending order). The correlation between two measurement occasions of the general factor decrease progressively as the lags of such measurements increase. Two consecutive measuring occasions

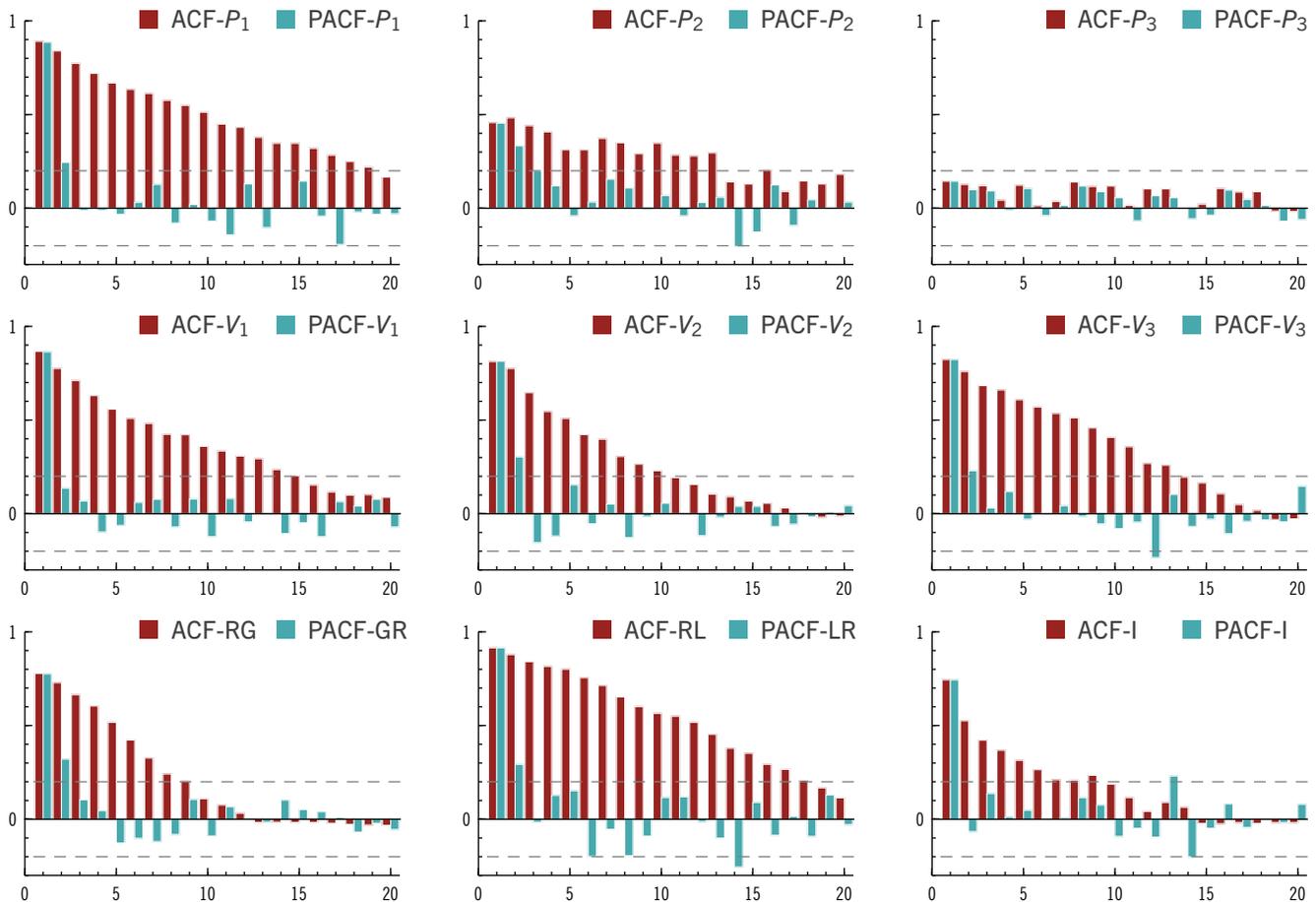


Figure 5. description of the auto-correlation factor (ACF) and partial auto-correlation factor (PACF) for each observable variable

In summary, Spearman's model (1904), the oldest one and the precursor of the factor studies in intelligence, is the model that explains properly the cognitive structure found in this study. It is interesting to note that when the intraindividual approach was used in a single case study, it returns to the model where it all began: Spearman's model.

The selection criterion of factor retention by the scree test can be questioned, since this criterion was essential for the evidence of a single factor. If another criterion was used, it could detect the presence of the three factors; in this case the CHC could be valid to explain the participant's performance. Two other criteria (eigenvalue greater than one and theoretical criterion) are commonly used to retain factors in an exploratory factor analysis. The first one determines the retention of factors that have an eigenvalue greater than one, being one of the most used criterion and a default procedure in popular statistical software, as the SPSS. Although being largely used, studies show that this criterion is inappropriate for small samples, i.e. less than 1,000 cases. The present study used a sample of 90 measurement occasions; therefore this criterion would not be appropriate. The second criterion usually employed is the theoretical one, in which the number of factors retained depends of the quantity of factors postulated theoretically. In this criterion, the appropriate number would be a solution with 3 factors.

Considering the possible argument that this study has not identified Gc, Gf and Gs only because of the retention criterion used, an exploratory factor analysis with three factors was conducted. The results did not indicate the presence of any of the broad abilities (Gf, Gc or Gs). Also, the three factors found in this particular analysis cannot be explained by any of the existing intelligence theories and models. Table 3 shows the tests factor loadings, by each factor. Factor 1 is strongly loaded by GR, I and P₃. Factor 2 is strongly loaded by V₂ and V₃ and factor 3 is strongly loaded by LR and P₁. The subject performance improved during the occasions and this may explain the factors. The tests LR and P₁ follow a similar path. Both indicate a gradual growth, but unstable, until half of the measurement occasions, when an asymptote is reached. However, P₂ loads moderately the factor 3 and does not represent this growth tendency. The test V₂ and V₃, which strongly loads factor 2, presents plateaus followed by a sudden change. The same pattern is shown in test V₁, but it present only moderately loads on factor 2. On the other hand, the P₂ test loads moderately the factor 2, but does not present the same growth tendency. The tests that better load factor 1 reach a quick growth, followed by a stable asymptote, until the end of the 90th measurement occasion. In sum, the three factors found can be interpreted as representing different growth trends.

However, it is not consistent with any traditional models of the interindividual approach and does not point to the validity of the CHC model, or the presence of Gf, Gs and Gc. The three factors presented a correlation range of .54 and .65, clearly indicating the occurrence of a general factor that has already been identified in the single factor solution.

To conclude, the single factor solution seems to be satisfactory to explain the participant's performance. All the tests present a good loading in the general factor, meaning they are well explained by the general factor. The general factor alone explains 67.28% of the participant's performance variance, while the solution with three factors explains 86.98% of the variance. Regarding the three factor solution, even if it had been chosen because of the theoretical criteria, it does not support the CHC model. However, the three factor solution brings new possibilities for future studies, taking into consideration the growth pattern of the participant's performance throughout the measurement occasions. New studies can investigate the empirical viability of these factors using a larger number of measurement occasions.

The results of the present study also suggest that population data cannot be used to make inference about single individuals' intelligence structure, a claim made several times both theoretically and empirically by Molenaar (2007A, 2007B). So we endorse that psychometric studies should expand their spectrum and not only invest in population based researches, but also in individual-based ones. Population and individual characteristics can coincide in some cases, but this does not seem to be the general rule. In order to better understand the complexity of the psychological processes and promote progress in the fields of Psychology and Psychometrics, it will be important to understand the convergences and divergences of these two approaches.

Future intraindividual studies should consider increasing the number of participants. The sample of this study was a single individual, who was a young adult, with normal development and with a University degree. Besides this limitation, the current study opens doors for an approach that is not commonly used in the psychometric literature on intelligence, and is used in a small scale in the Psychological literature worldwide. Future studies, with larger and more diverse samples, will be able to contribute for a better understanding about the different patterns of the individuals' cognitive architecture even though we understand that there may be some important obstacles to the intraindividual approach study. This type of study requires multiple measurement occasions and the availability of people to participate in such studies might be limited. A potential way to address this limitation is to use new technologies of data collection, such as smartphones and other web-related technologies. One good example of using new technologies to collect a large amount of data from individuals is the *Flu Near You* project (see: <https://flunearyou.org/>), which uses a smartphone's app to collect flu-related symptoms in a day-by-day fashion. In the same line, Van de Leemput et al. (2014) studied time series (200 assessment occasions) of four emotions in a large group of healthy and depressed people through an app on a smartphone. The authors point that nowadays is easier to assess and monitor psychological variables, such as mood indicators, in individuals due to the advancement of web applications (Van de Leemput et al., 2014). They provide a user-friendly interface in a tool (smartphone) that is used several times per day, and also enables the implementation of assessment feedback which may

Table 3. factors and factor loadings

	factor 1	factor 2	factor 3
P ₁	0.20	0.08	0.78
P ₂	-0.06	0.40	0.45
P ₃	0.71	0.11	-0.10
V ₁	0.31	0.39	0.43
V ₂	0.23	0.91	-0.09
V ₃	-0.17	0.89	0.15
GR	0.91	-0.03	0.13
LR	0.02	-0.03	0.94
I	0.74	0.07	0.13

increase the motivation of the participant to answer the research questions several times. Finally, another outstanding example of massive intraindividual data collection was provided by Myin-Germeys, Oorschot, Collip, Lataster, Delespaul and van Os (2009).

The current study used the intraindividual approach to evaluate a cognitive model and the instrument used was a particular battery.

Further studies could be conducted throughout the observation of specific behaviors, for example, during a certain period of time, as the above mentioned papers. Researchers that focus in the behavior analysis have already the expertise to observe and register behaviors and the intraindividual approach can bring them a contribution in relation to the data analysis. ■

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